Pulse Localization and Fourier Analysis in the Mathematical Model of Acoustic Transient Field

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DOI: 10.15598/aeee.v14i3.1433

Abstract. *The numerical model of a semi-cylindrical acoustic diffuser in planar transient acoustic field is discussed. The finite element method was used for the solution of the model. From the computed waveforms the straight and the reflected pulses were automatically extracted using cross-correlation. The harmonic decomposition was performed on the obtained pulses and the results were plotted in the polar pattern.*

Keywords

Acoustic diffuser, cross-correlation, numerical modelling, room acoustics.

1. Introduction

Diffusers are devices that scatter matter or energy [1]. In acoustics, this scattering is based on a reflection of an incident acoustic wave from irregular surface of a diffuser's body. The probabilities of reflections, in the case of the ideal diffuser, are equal in all directions and the scattering is uniform. Room acoustics uses this feature for making an ambient sound field in the room. There are no focal points in such room and the field is without standing waves. Another use of the diffusers is scattering large amount of energy produced, e.g. by transportation.

In this paper numerical modelling of semi-cylindrical acoustic diffuser placed in the anechoic chamber is carried out, and processing of obtained waveforms is presented. The aim is to obtain a response of acoustic pressure generated by an acoustic diffuser placed in the room, which is a typical task in the field of room acoustic. Similar problems are dealt with by many acoustic research groups in the world, e.g. in [2], [5], [6], [9] and [13], but the authors present innovative approach aimed to localization of the reflected wave from the diffuser using the cross-correlation.

2. Mathematical Model

For the purpose of obtaining acoustic pressure field around the diffuser the corresponding wave equation [4] was solved in time domain [8] and [11]. The derivation of this equation starts from:

- The Newton's law of motion in the form of a highly simplified Navier-Stokes equation for compressible fluids with neglected viscosity, external forces on the particles and velocity which is considered small:
$$\varrho \frac{\partial \vec{v}}{\partial t} = -\operatorname{grad} p\,. \tag{1}$$
- The continuity equation that represents the law of conservation of matter in the differential form:
$$\operatorname{div}(\varrho \vec{v}) + \frac{\partial \varrho}{\partial t} = 0\,. \tag{2}$$
- The ideal gas law (the Poisson law) for the adiabatic processes:
$$p\varrho^{-\kappa} = C\,. \tag{3}$$

In the above equations, ϱ denotes the specific mass, symbol $\vec{v}$ stands for the acoustic velocity, t represents time, p is the acoustic pressure, κ denotes the ratio of specific heats for the particular continuum (for air $\kappa = 1.4$) and, finally, C is a general constant.

The resulting wave equation is in the form of:

$$-\operatorname{div}\left(\frac{1}{\varrho}\operatorname{grad} p\right) + \frac{1}{\varrho c^2} \cdot \frac{\partial^2 p}{\partial t^2} = 0\,, \tag{4}$$

where ϱ denotes the mass density, p is the acoustic pressure, c stands for the speed of sound in gas at the standard conditions and t represents time.

In order to determine similarity of two series f and g, cross-correlation can be performed, Eq. (5). In the case of very similar signals, the cross-correlation returns a high peak in time, where both signals are in the same phase. The SciPy.signal Python library [10] implementation was used in this research:

$$R(k) = \sum_{i=-\infty}^{\infty} f(i) \cdot g(k+i-n)\,, \qquad (5)$$

where k is time-lag between the series f and g and the symbol n stands for the number of samples.

For harmonic decomposition of a discrete signal of N samples the Discrete Fourier transform is usually used [4]:

$$S(k/NT) = \sum_{n=0}^{N-1} s(nT) \exp\left(\frac{j2\pi kn}{N}\right), \qquad (6)$$

where $k/NT = k\Omega$ is the spectral line number, nT is discrete time and $T = \frac{1}{f_s}$ is the sampling period.

3. Illustrative Example

Consider a semi-cylindrical acoustic diffuser in an anechoic chamber (for physical dimensions see Fig. 1). In front of the diffuser there is a pressure source transmitting a Gaussian pulse with 1 kHz bandwidth. The advantage of the Gaussian pulse is flat spectral density within the bounds of the bandwidth and rapid drop beyond. Measuring points (probes) are evenly placed around the diffuser.

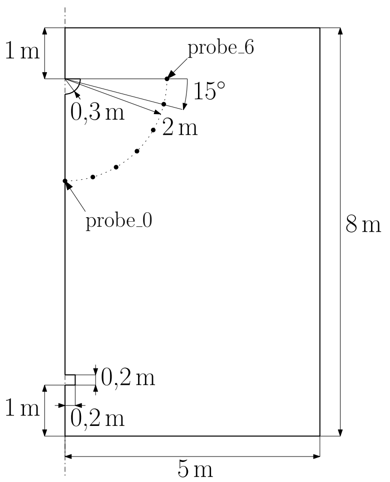

Fig. 1: Physical arrangement of the example.

4. Numerical Solution

The wave Eq. (4) was solved in time domain and in 2D axisymmetric arrangement using Comsol Multiphysics [1] and Agros2D application [7] and [12]. The model definition is depicted in Fig. 2. The wave equation is computed in the air with the given mass density $\varrho = 1.2\ \mathrm{kg{\cdot}m^{-3}}$ and speed of propagation $c = 343\ \mathrm{m{\cdot}s^{-1}}$. The air is surrounded by boundary conditions:

- Matched boundary–the Neumann condition simulating absorptive characteristic of the anechoic chamber.
- Diffuser–the Neumann condition simulating absolutely reflective (sound-hard) surface of the diffuser.
- Axis of symmetry–the Neumann condition used for reduction of degrees of freedom in the model.
- Pulse source–the Dirichlet condition acting as a source of time varying acoustic pressure.

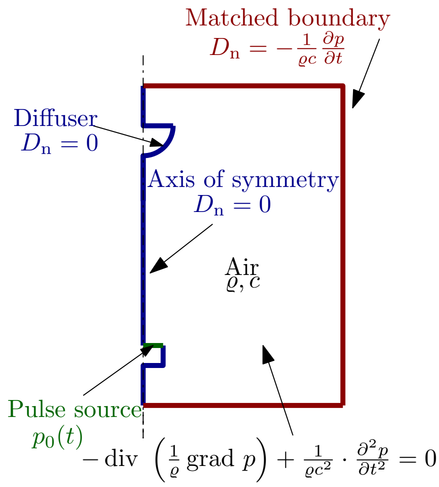

Fig. 2: Definition of the numerical model.

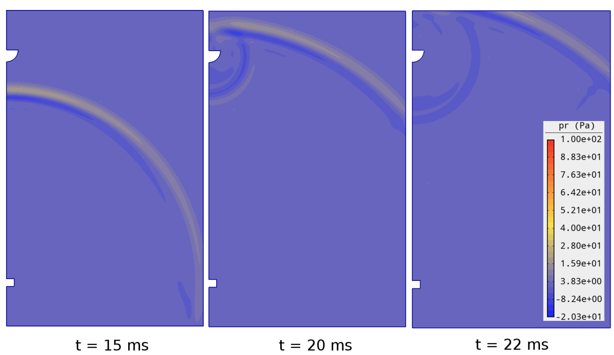

Fig. 3: Acoustic pressure around diffuser.

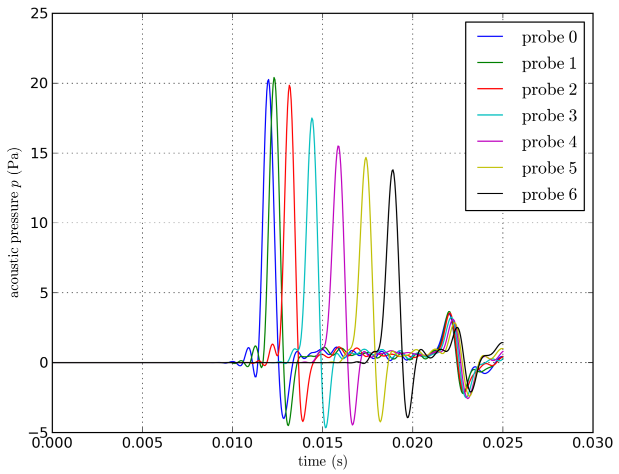

Fig. 4: Acoustic pressure in the measuring points.

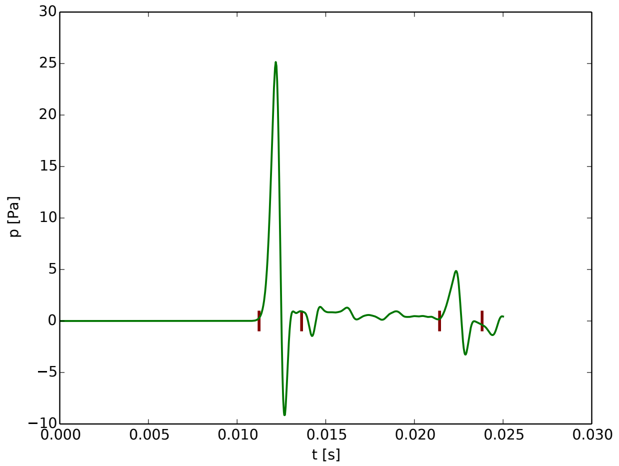

Fig. 5: Successful localization of the incident and reflected wave (acoustic pressure in front of the diffuser).

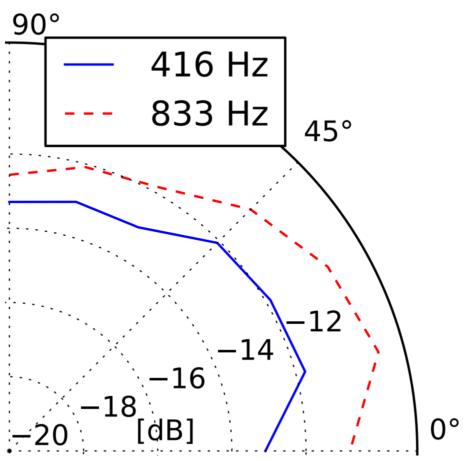

Fig. 6: Polar pattern of the diffuser.

The elements covering the area are of the polynomial order of 2 and their maximum surface was set to 0.03 m^2. The transient was calculated for time ranging from 0 s to 0.025 s with the length of the fixed time step $\Delta t = 3.125 \cdot 10^{-5}$ s.

The solution of model above in several time steps is shown in Fig. 3. The propagation of the sound wave and also the reflection on the diffuser can be seen in this picture. The acoustic pressure captured in seven equidistant points (probes; see Fig. 1) in front of the diffuser is drawn in Fig 4.

4.1. Postprocessing

Based on the previous knowledge of supposed pulse waveform, cross-correlation Eq. (5) was used to localize the desired parts of the signal. The mathematical expression of the Gaussian pulse was used as the reference signal which replicas in the computed acoustic pressure were sought for. With this method it is possible to find the centre of both incident and reflected pulse in the whole waveform automatically. The acoustic pressure at the point in the axis of symmetry with the localized pulses marked out can be seen in Fig. 5.

When the incident and reflected waveform are isolated from the whole sequence, it is possible to perform their harmonic decomposition using Discrete Fourier transform Eq. (6), with the sampling rate of 2 kHz. The double sampling frequency compared to the excitation signal was chosen with respect to the computational requirements. Then, performing this action on waveforms obtained from all measuring points, the ratio of amplitudes of the corresponding frequencies can be plotted into the polar pattern. The resulting diagram can serve for a quick evaluation of the reflective characteristics of the scattering element. An example with two frequencies is in Fig. 6, where 0° corresponds to the measuring point in the axis of symmetry.

5. Conclusion

The knowledge of time evolution of acoustic pressure reflected from the diffuser is crucial for the design and evaluation of function of this diffuser shape, and numerical modelling represents a tool for this evaluation much cheaper than measurement.

From the acquired acoustic pressures we can easily obtain graphical characteristics of the scattering properties of the diffuser useful for quick determination of its qualities.

Acknowledgment

This work was supported by the Ministry of Industry and Trade of The Czech Republic under the project No. FR-TI4/569: Development of a new generation of acoustic diffusers and their modelling, and University of West Bohemia project SGS-2012-039.

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